Results

Using the above noniterative method, solutions of Eqs. (1–4) were calculated for a wide range of values of the parameters. For comparison purposes, solutions of Eqs. (1–4) computed by the method of transformation groups as given in Ref. 1 are compared with results obtained by the present method. Selected results are shown in Table 1. The high degree of accuracy depicted is typical of that obtained by using this simple and straightforward method.

Table 1 Comparative results

		$\varepsilon = 0.50$				$\varepsilon = 0.75$	
α	ρ	λ(Ref. 1)	λ(Present)	α	ρ	λ(Ref. 1)	λ(Present)
40°	0.5180	0.8051	0.8012	40°	0.5618	0.9467	0.9440
	0.2772	0.6914	0.6900		0.3145	0.8899	0.8892
	0.2260	0.6641	0.6634		0.2594	0.8751	0.8747
	0.1951	0.6471	0.6469		0.2256	0.8656	0.8655
	0.1789	0.6381	0.6380		0.2072	0.8603	0.8603
80°	0.5284	0.8377	0.8347	80°	0.4598	1.0570	1.0560
	0.3321	0.7722	0.7710		0.3393	1.0358	1.0353
	0.2606	0.7469	0.7464		0.2808	1.0251	1.0249
	0.2068	0.7274	0.7271		0.2448	1.0184	1.0183
	0.1901	0.7212	0.7211		0.2247	1.0146	1.0146

References

[1] Kurajian, G. M. and Na, T. Y., "An Initial Value Method for the Solution of an Eigenvalue Problem in Solid Mechanics," *Transactions of the ASME*: Ser. E, *Journal of Applied Mechanics*, Vol. 39, No. 3, Sept. 1972, pp. 854–855.

[2] Oden, J. T. and Childs, S. B., "Finite Deflections of a Nonlinearly Elastic Bar," *Transactions of the ASME*: Ser. E: *Journal of Applied Mechanics*, Vol. 37, No. 1, March 1970, pp. 48–52.

[3] Rubbert, P. E. and Landahl, M. T., "Solution of Nonlinear Flow Problems Through Parametric Differentiation," *Physics of Fluids*, Vol. 10, No. 4, April 1967, pp. 831–835.

[4] Tan, C. W. and DiBiano, "A Parametric Study of Falkner-Skan Problem with Mass Transfer," *AIAA Journal*, Vol. 10, No. 7, July 1972, pp. 923–925.

[5] Narayana, C. L. and Ramamoorthy, P., "Compressible Boundary-Layer Equations Solved by the Method of Parameter Differentiation," *AIAA Journal*, Vol. 10, No. 8, Aug. 1972, pp. 1085–1086.

[6] Na, T. Y., "Comment on 'Compressible Boundary–Layer Equations Solved by the Method of Parameter Differentiation'," *AIAA Journal*, Vol. 11, No. 12, Dec. 1973, pp. 1790–1791.

[7] Na, T. Y. and Habib, I. S., "Non-Iterative Solution of Natural Convection Equations by Parameter Differentiation," *International Journal of Heat and Mass Transfer*, Vol. 17, No. 3, March 1974, pp. 253–254.

Entropy Layer on a Supersonic Plane Flat Nose at Incidence

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Nomenclature

d = model depth and reference length
n = probe station
p_o, T_o = freestream stagnation pressure, temperature
w = probe depth
α = model incidence
γ = ratio of specific heats
θ = probe angle
λ = total pressure ratio, probe to freestream

Subscript
m = value of θ, λ for max pitot pressure recovery

Received March 26, 1974; revision received July 22, 1974. This work is supported by Australian Research Grants Committee.
Index category: Supersonic and Hypersonic Flow.
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Introduction

THE calculations of Swigart[1] and Webb et al.,[2] using an inverse method, predicted the existence of an entropy layer for blunt round axisymmetric bodies at incidence, wherein the maximum entropy does not wet the body surface. The thickness of the presumed layer would however be very small and difficult to detect for such bodies. The method of integral relations has also been used for blunt body calculations by Prosnak[3] on the one hand, who ignored the layer, and by others[4–6] who did not ignore the entropy layer but assumed a certain stagnation streamline shape. The time dependent computational method recently developed by Moretti[7–9] and Godunov's finite difference method[10] do not contain such a restraint, but their use has been confined to round nosed symmetric shapes. Furthermore, there does not appear to have been published any experimental evidence for such an entropy layer on either plane or axisymmetric blunt bodies. The plane flat plate with sharp corners, when placed in an extreme asymmetric attitude should lend itself well to a study of this question, provided that the entropy layer extends well into the inviscid region (Fig. 1).

Design of Experiment

On the assumption that both the entropy layer and boundary layer are thin compared with the shock layer thickness of about $\frac{1}{4}$ in. (6 mm) on the flat faced block [$d = \frac{3}{4}$ in. (19 mm)] for the conditions of the test, it was presumed that the streamlines of interest would be substantially parallel to the face as they passed the corner. The tests were performed in a $4 \times 5\frac{1}{2}$ in. (102×140 mm) test section at a Mach number $M = 3.08$ and a unit Reynolds number of 10^6/in. (4×10^7 per m). The freestream stagnation pressure and temperature were $p_0 = 75$ psia (517 *kPa*) and $T_0 = 520$ R (288 K).

Provided that the boundary–layer thickness remains small compared to that of the entropy layer, a total head probe traverse at the corner might be expected to reveal the true total pressure and hence entropy variation. The laminar boundary layer at the corner is estimated to be about 0.001 in. (0.025 mm) thick, for the conditions of this experiment. Thus, a carefully designed traverse mechanism having a read out accuracy of this order or better could detect the presence of an entropy layer thicker than about 0.0025 in. (0.06 mm). The accuracy claimed for this experiment is in fact of the order of 1/10 of this value.

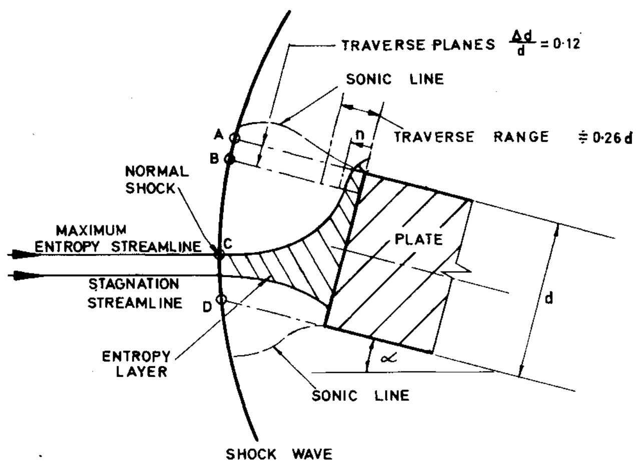

Fig. 1 Flat nosed plate at incidence in supersonic flow.

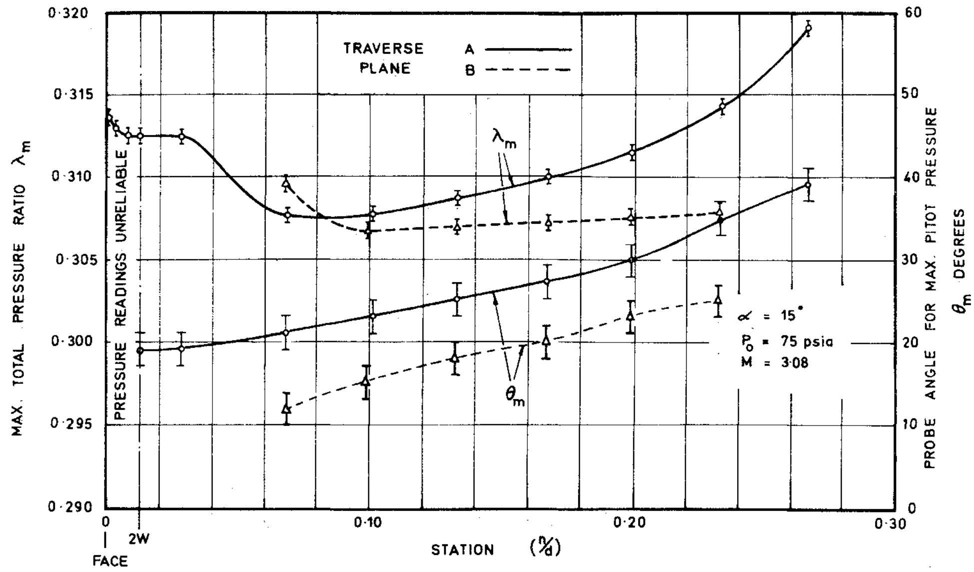

Fig. 2 Total pressure and flow angle distribution near corner.

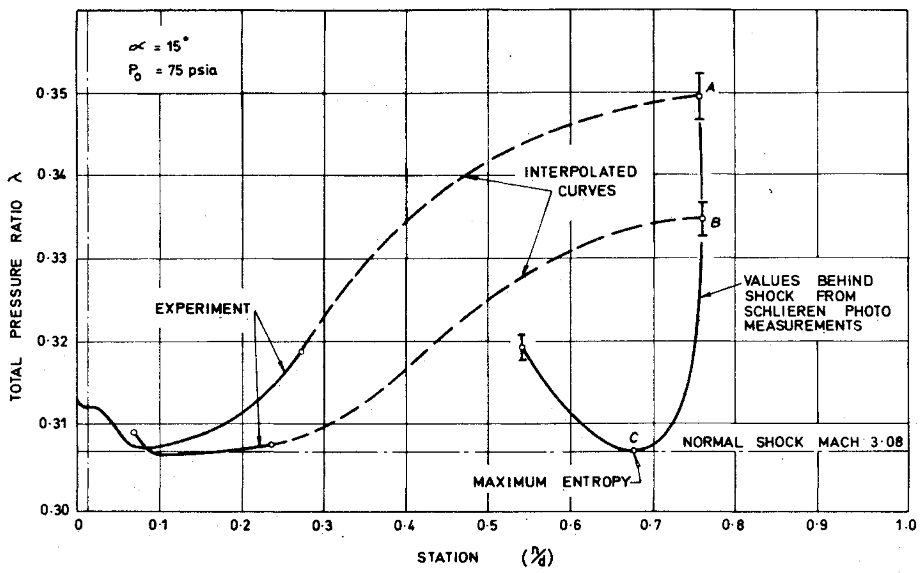

Fig. 3 Total pressure distribution from model face to shock.

A small pitot probe made from flattened 1 mm diam tubing was traversed over a range of 0.20 in. (5 mm) normal to the face from the model surface near the corner (Fig. 1). According to Ref. 11, Reynolds number effects due to small probe size on the total pressure reading will be less than 1% under the conditions of this experiment, and the flow displacement effect, due to the relatively large size of the probe (0.005 in., 0.127 mm) compared with the length of the traverse (0.20 in., 5 mm), can be neglected.

Based upon the data of Ref. 11 for round probes, probe-model interference for a flattened probe was assumed to be negligible when the probe was more than two times the probe depth (w) away from the face of the model. On the other hand, results obtained when the probe was closer than this to the model were considered unreliable.

The probe pitot pressure was calibrated against inclination angle θ in an air jet from a 1 in. (25.4 mm) diam nozzle. With the model set at 15° incidence, traverses were made in the two planes A and B, (Fig. 1), with the probe axis inclined at 0°, 10°, 20°, 30°, and 45° to the front face of the model. Thus it was possible to find the max pressure recovery, λ_m, at each position. The corresponding probe angles θ_m were also obtained as supplementary information, but the precise relationship of probe to local flow angle in this particular flowfield is not known. Results for the max total pressure distribution and corresponding

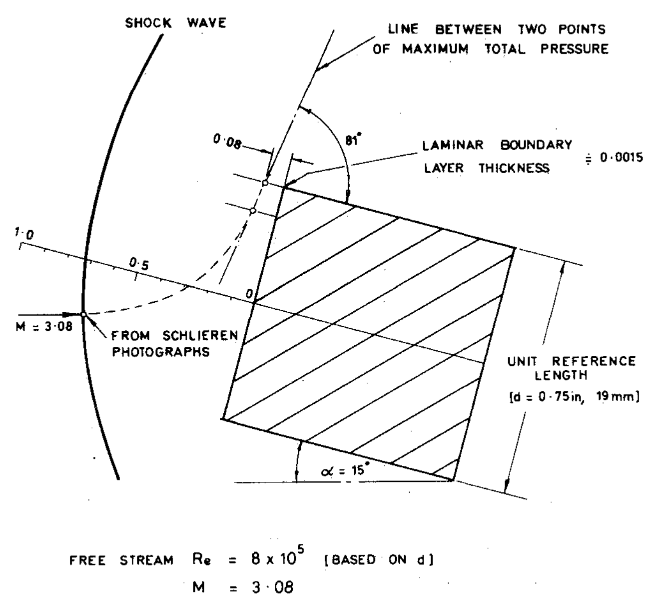

Fig. 4 Maximum entropy streamline for flat nosed plate at incidence (to scale).

probe angle are given in Fig. 2, and their relation to the whole flowfield is shown in Fig. 3. At the shock, stagnation pressure ratio was determined from the local shock angle obtained from a Schlieren photograph. The angles were measured by fitting a circular arc to the relevant portion of the shock, with the indicated over-all accuracy. The approximate location of the maximum entropy streamline between the shock and the edge of the model could therefore be reconstructed, Fig. 4.

Conclusion

Experimental evidence has been obtained for the location of the max entropy streamline behind the detached curved shock on a supersonic two-dimensional flat face. The results support the idea of an entropy layer for such blunt bodies in which the maximum entropy streamline does not wet the body surface.

References

[1] Swigart, R. J., "Hypersonic Blunt Body Flow Fields at Angle of Attack," *AIAA Journal*, Vol. 2, No. 1, Jan. 1964, pp. 115–117.

[2] Webb, H. G. Jr., Dresser, H. S., Adler, B. K., and Waiter, S. A., "Inverse Solution of Blunt-Body Flowfields at Large Angle of Attack," *AIAA Journal*, Vol. 5, No. 6, June 1967, pp. 1079–1085.

[3] Prosnak, W. J., "The Asymmetric Hypersonic Blunt Body Problem," *Fluid Dynamic Transactions*, Vol. 2., Pergamon Press, New York, 1965, pp. 457–476.

[4] Belotserkovskii, O. M. and Chushkin, P. I., "The Numerical Solution of Problems in Gasdynamics," *Basic Developments in Fluid Dynamics*, Vol. 1, Academic Press, New York, 1965, pp. 55–73.

[5] Archer, R. D., "Fast Solution to Supersonic Plane Flat Faced Blunt Body," *AIAA Journal*, Vol. 10, No. 5, May 1972, pp. 707–708.

[6] Brong, E. A. and Leigh, D. C., "Method of Belotserkovskii for Asymmetric Blunt Body Flows," *AIAA Journal*, Vol. 2, No. 10, Oct. 1964, pp. 1852–1853.

[7] Moretti, G. and Abbott, M., "A Time-Dependent Computational Method for Blunt Body Flows," *AIAA Journal*, Vol. 4, No. 14, Dec. 1966, pp. 2136–2141.

[8] Moretti, G. and Bleich, G., "Three-Dimensional Flow around Blunt Bodies," *AIAA Journal*, Vol. 5, No. 9, Sept. 1967, pp. 1557–1562.

[9] D'Souza, N., Molder, S., and Moretti, G., "Numerical Method for Hypersonic Internal Flow over Blunt Leading Edges and Two Blunt Bodies," *AIAA Journal*, Vol. 10, No. 5, May 1972, pp. 617–622.

[10] Masson, B. S., Taylor, T. D. and Foster, R. M., "Application of Godunov's Method to Blunt Body Calculations," *AIAA Journal*, Vol. 7, No. 4, April 1969, pp. 694–698.

[11] Bryer, D. W. and Pankhurst, R. C., *Pressure-Probe Methods for Determining Wind Speed and Flow Direction*, National Physical Lab., England, 1971.

Calculation of Turbulent Shear Stress in Supersonic Boundary-Layer Flows

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Nomenclature

C_f = skin friction coefficient
M = Mach number
P = pressure
r = distance normal to centerline
R = radius of the duct
T_{ij} = total stress tensor
u = time-averaged velocity in primary flow direction
v = time-averaged velocity normal to centerline
x = distance parallel to centerline
y = $R-r$
δ = boundary layer thickness
μ = molecular viscosity
η = y/δ
ρ = time-averaged density
τ = shear stress

Subscripts

e = boundary-layer edge condition
∞ = freestream condition

Superscript

$\langle(\ )\rangle'$ = time-averaged fluctuation value

Received March 29, 1974; revision received June 21, 1974. This work was supported by NASA Grant NGR-48-002-047 under administration of the Aerodynamics Branch, NASA Ames Research Center.

Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Supersonic and Hypersonic Flow; Shock Waves and Detonations.

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Introduction

IN the study of supersonic turbulent boundary-layer flow the turbulent shear stress distribution has always been of great importance and interest. The direct measurement of the turbulent shear stress is, however, quite difficult. A natural alternative is to compute the shear from experimental mean flow data by numerically integrating the momentum equation. Such computations have been performed in recent studies by Bushnell and Morris,[1] Horstman and Owen,[2] and Sturek.[3] This Note describes results obtained by a computational procedure which differs from those previously reported in that integrated mass and momentum flux profiles, and differentials of these integral quantities are used in the computations so that local evaluation of the streamwise velocity gradient is not necessary. The computed results are compared with measured shear stress data obtained by using hot wire anemometer and laser velocimeter techniques in recent studies by Rose and Johnson.[4,5] The measurements of Rose and Johnson were made upstream and downstream of an adiabatic unseparated interaction of an oblique shock wave with the turbulent boundary layer on the flat wall of a two-dimensional, $M_\infty = 2.9$ wind tunnel. The shock wave was generated by a 7° wedge. The turbulence data obtained from the two independent systems of measurement were in reasonably good agreement, indicating that the data should be reliable. The computational procedure developed here is easy to use, and the computed results show reasonably good over-all agreement with those obtained by direct measurement. As would be expected for any method of computing shear stress from mean flow data, the computed values of shear stress are quite sensitive to small differences in mean flow profiles and to simplifying assumptions which may be made in developing the relationships to be used in the computations. The effect of some of these differences on computed shear stress distributions is discussed.

Basic Equations and Boundary Conditions

The time-averaged equations for the conservation of mass and momentum for steady compressible turbulent boundary-layer flow in an axisymmetric channel are, respectively,

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial x}\langle\rho' u'\rangle+\frac{1}{r}\frac{\partial}{\partial r}(r\rho v)+\frac{1}{r}\frac{\partial}{\partial r}(r\langle\rho' v'\rangle)=0 \qquad (1)$$

and

$$\frac{\partial}{\partial x}(\rho u^2)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho uv)=-\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}T_{xx}+\frac{1}{r}\frac{\partial}{\partial r}rT_{rx} \qquad (2)$$

where

$$T_{xx}=(\tau_v)_{xx}-(\rho\langle u'^2\rangle+2u\langle\rho' u'\rangle) \qquad (3)$$

$$T_{rx}=(\tau_v)_{rx}-(\rho\langle u'v'\rangle+u\langle\rho' v'\rangle+v\langle\rho' u'\rangle) \qquad (4)$$

with τ_v representing the viscous stress.

If we assume that $|v\langle\rho' u'\rangle|\ll|\rho\langle u'v'\rangle|$ and $|\partial\langle\rho' u'\rangle/\partial x|\ll|\partial(\rho u)/\partial x|$ and transform to an x-y coordinate system, the